BEAM PROPAGATION METHOD APPLIED TO SINGLE
AND DOUBLE STEP DISCONTINUITIES IN PLANAR
OPTICAL WAVEGUIDES

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Abstract

The power transmission and loss at a discontinuity in
planar optical waveguides are calculated numerically using
the Beam Propagation Method (BPM) for transverse elctric
modes. A single step discontinuity including changes in
core thickness and refractive index are considered as well
as symmetric double step discontinuity. A comparison
between the results obtained by this method and those
obtained by three other methods shows a good agreement.

Introduction

The powers transmitted and lost at the junction
between two nonidentical planar optical waveguides can be
calculated by means of many well known techniques:
variational[1], mode matching[2], Wiener-Hopf[3], residue
calculus[4] and Green's function[5]. All of these methods
are relatively complicated and usually require the solution
of an infinite system of equations or the expansion of the
electromagnetic field in terms of an infinite set of
orthogonal functions or polynomials which are oscillatory,
and this requires a great care to guarantee the stability
and the convergence of the solution.

In many practical integrated optical devices, we need
to fabricate two or more optical waveguides with different
optogeometric properties on the same substrate. At the
junction between such waveguides, there exist transmitted
and reflected fields. Fortunately, in many practical
cases, these nonidentical waveguides have slightly
different optogeometric properties: small variations in
the refractive index and the waveguide's thickness. This
allows us to neglect the reflected field. In such cases
the BPM[6],[7] can be used efficiently to calculate the
transmitted field through a junction between two
nonidentical waveguides. Of course the method can be
applied to optical waveguides with two-dimensional
confinement of the light power like optical fibers, but in
this case a two dimensional Fourier transform is required
instead of the one-dimensional transform needed for planar
waveguides.

Theory

The BPM consists basically of propagating the electromagnetic field over a small distance Δz through a homogeneous medium and then correcting for the refractive index variations during the propagation distance Δz.

Let us assume that $E_y(x,z)\hat{y}$ is the propagating electric field in a y-invariant inhomogeneous planar optical waveguide, where $\hat{y}$ is a unit vector in the y-direction. This field is the solution of the scalar wave equation:

$$\frac{\partial^2 E_y(x,z)}{\partial z^2} + [\nabla_t^2 + k_o n^2(x)] E_y(x,z) = 0 \quad \text{-----------(1)}$$

where ∇_t^2 is the transverse laplacian $\partial^2/\partial x^2$, k_o is the free space wave number, and $n(x)$ is the refractive index distribution in the direction transverse to the direction of propagation z. The time dependence is assumed to be $\exp(i\omega t)$, where ω is the radial frequency of the electromagnetic field and $i=\sqrt{-1}$. The solution of (1) at $z=\Delta z$ may be written formally in terms of the field at $z=0$ as follows[6]:

$$E_y(x,\Delta z) = e^{-i\Delta z[\nabla_t^2 + k_o^2 n^2(x)]^{1/2}} . E_y(x,0) \quad \text{--------------(2)}$$

The square root in (2) may be written as:

$$[\nabla_t^2 + k_o^2 n^2(x)]^{1/2} = \frac{\nabla_t^2}{[k_o^2 n^2(x) + \nabla_t^2]^{1/2} + k_o n(x)} + k_o n(x) \text{----(3)}$$

If $n(x)$ in the first right-hand member of (3) is replaced by a certain constant value n_s (typically the substrate index of refraction), an approximation to (2) takes the form:

$$E_y(x,\Delta z) = e^{-i\Delta z\{[\nabla_t^2/(\nabla_t^2+k_o^2 n_s^2)^{1/2}+k_o n_s]+k_o[n(x)-n_s]\}} . E_y(x,0) \text{--(4)}$$

Each squared braket in the exponential of (4) is an operator; these two operators do not commute, but to the second order in Δz, equation (4) can be written as[6]:

$$E_y(x,\Delta z) = P.Q.P \{E_y(x,0)\} \quad \text{-------------------------(5)}$$

where P and Q are the two operators:

$$P = e^{-i\frac{\Delta z}{2}\{\nabla_t^2/[(\nabla_t^2 + k_o^2 n_s^2)^{1/2} + k_o n_s]\}} \quad \text{---------------(6)}$$

and

$$Q = e^{-i \frac{\Delta z}{2}\{k_o[n(x)-n_e]\}} \quad \text{---------------------} (7)$$

Operating with P on $E_y(x,0)$ is simply a propagation of the
field in a homogeneous medium (whose index is n_e) over a
distance $\Delta z/2$. The operator Q accounts for the variations
in the index of refraction $n(x)$. The error introduced in
the solution (5) is proportional to $(\Delta z)^3$ and hence small
propagating steps are necessary to obtain well accurate
results [7]. The direct operation of P on $E_y(x,0)$ involves
the expansion of the exponential operator (6) in an
infinite series of differential operators which are
extremely difficult to treat in numerical applications,
however if we consider the spatial Fourier transform of
$E_y(x,0)$, the propagation over $\Delta z/2$ is reduced to a simple
multiplication of the transform of $E_y(x,0)$ and the
transform of the operator P which is simply [6]:

$$F\{P\} = e^{i \frac{\Delta z}{2}\{k_t^2 / [(k_t n_e - k_t^2)^{1/2}+k_o n_e]\}} \quad \text{------------} (8)$$

where $F\{P\}$ denotes the Fourier transform of P and k_t is the
variable of the transform. Although in the BPM we deal with
the total propagating field regardless of its modal
content, we can restore easily the modal behaviour of
$E_y(x,z)$ through the eigenmode expansion [6]:

$$E_y(x,z) = \sum_n t_n e_{yn}(x) e^{-i\beta_n z} + R \quad \text{------------} (9)$$

where $e_{yn}(x)$ is the transverse distribution of the
electromagnetic field of the nth guided mode propagating in
the z-direction with a propagation constant β_n. The
radiation field in the eigenmode expansion (9) has the form
of a Fourier integral which is denoted by R. The constant
t_n is the expansion coefficient of the nth guided mode of
the waveguiding structure. This coefficient can be
calculated by taking the scalar product of (9) with the
complex conjugate $e_{yn}^*(x)\exp(i\beta_n z)$:

$$t_n = \frac{\int_{-\infty}^{\infty} E_y(x,z).e_{yn}^*(x)\, dx}{\int_{-\infty}^{\infty} |e_{yn}(x)|^2\, dx} \quad \text{-----------------} (10)$$

The power P_n carried by that mode is simply:

$$P_n = \frac{|t_n|^2 \beta_n}{2 \omega \mu_o} \int_{-\infty}^{\infty} |e_{yn}(x)|^2\, dx \quad \text{-------------------} (11)$$

The propagation of the total field from z to z+ Δz is performed through three steps: the operator P (the propagator) is applied on $E_y(x,z)$ using the Fourier transform; the resulting field is then operated on by Q (the corrector) giving rise to the corrected field, finallly the corrected field is operated on by P to obtain the total field at z+ Δz. This process can be continued until the desired propagation distance is reached.

A symmetric single step discontinuity

A single step discontinuity consisting of two butt-joined nonidentical symmetric waveguides is shown in figure 1. This case was studied previously by Marcuse[9] using an approximate mode matching technique, Ittipiboon et.al.[4] studied the same case using residue calculus technique, also Nishimura et.al.[10] used an approximate integral equation formulation to calculate the relative radiated power P_r (the ratio between the power lost by radiation and the power carried by the incident mode from the left of the step). Figure 1 shows the results of the BPM (solid dots) and those of the three previous techniques, where P_r is plotted as a function of $k_o d_1$ when d_2/d_1 is kept constant equal to 0.5 and the wavelength is equal to 0.6328 µm.

An asymmetric single step

The second case we considered was an asymmetric step which was studied rigorously by Boyd et. al.[11] using a discrete representation of the radiation field. Figure 2 shows the magnitude of the transmission coefficient $|t_n|$ of the fundamental mode (i.e. n=0) as a function of $k_o d_1$ when the step ratio d_2/d_1=0.5 at a wavelength equal to 0.6328 micron. The solid dots represent the results of the BPM.

The fairly good agreement between the results of the BPM and the other methods is due to the fact that the forward radiated power is much higher than the reflected guided and backscattered radiated power for a step ratio equal to 0.5 as pointed out by Marcuse[9]. This means that the losses are mainly due to forward scattering; and this justifies the fundamental approximation used in the BPM: the reflected field can be neglected.

Double step

It is well known that a dielectric optical waveguide allows, besides a finite number of guided modes, a continuum of modes. The modes within a finite range of the continuum are propagating; the rest represent energy stored locally in the neighborhood of the discontinuity (non propagating modes). In the double step shown in figure 3, the input waveguide (to the left of the step S1) is single

mode and identical to the output waveguide (to the right of
the step S2); while the intermediate guide of length L
allows the first two even guided TE modes to propagate.
When the fundamental mode of the input guide is incident
from the left it is scattered by the step S1 in all guided
modes allowed at either side of S1 as well as in the
continuous spectrum of modes. After propagating away from
the step S1, the guided modes and the propagating part of
the continuous spectrum are again scattered by the second
step S2, so that interference between the fields scattered
by S1 and S2 takes place: we say that the propagating part
of the continuous spectrum of modes excited at one step
"sees" the adjacent step. On the other hand, the phase of
the transmitted guided mode at the output waveguide depends
on the length L of the intermediate guide. These
interferences influence the guided transmitted field at the
output waveguide and the transmission coefficient $|t_o|$ of the
fundamental mode at the output waveguide exhibits
oscillations as function of L; this phenomena was pointed
out by Rozzi[12], Biehlig[13] and Koshiba et.al.[14].
Figure 3 shows the results obtained by the BPM, where the
period of the oscillation of t_o is approximately equal to 80
microns.

Conclusion

To the author's knowledge, the BPM applied to the
problem of waveguide discontinuity is presented for the
first time in this paper. The agreement between the results
obtained by the BPM and those obtained by other methods is
fairly good. The ability to treat a phase-sensitive
discontinuity using the BPM is proved by considering a
double step. We think that the BPM is one of the powerful
and simple methods that can deal with optical waveguide
discontinuities, and may be it will be the most convenient
and the simplest method for studying the discontinuities in
waveguides having arbitrary refractive index distributions.

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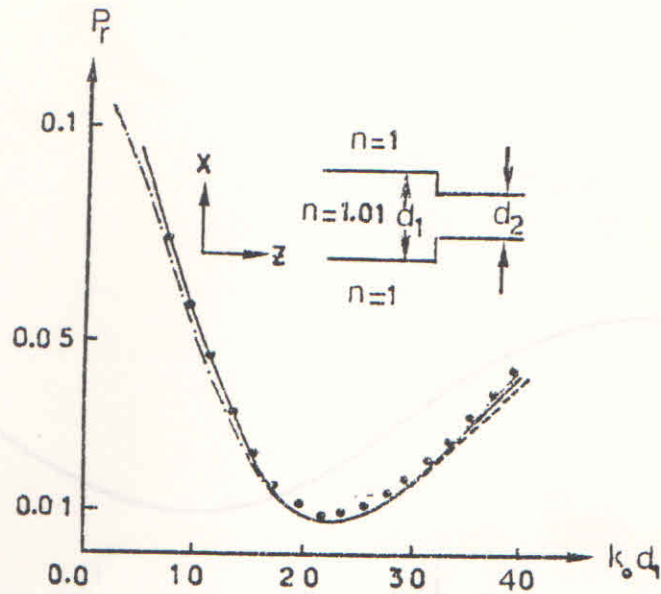Trans. IECE Japan, vol. E 66, No.4, pp.250-251, 1983.

Figure 1 – A symmetric step discontinuity:
—————— Integral equation method
—·—·— Residue calculus technique
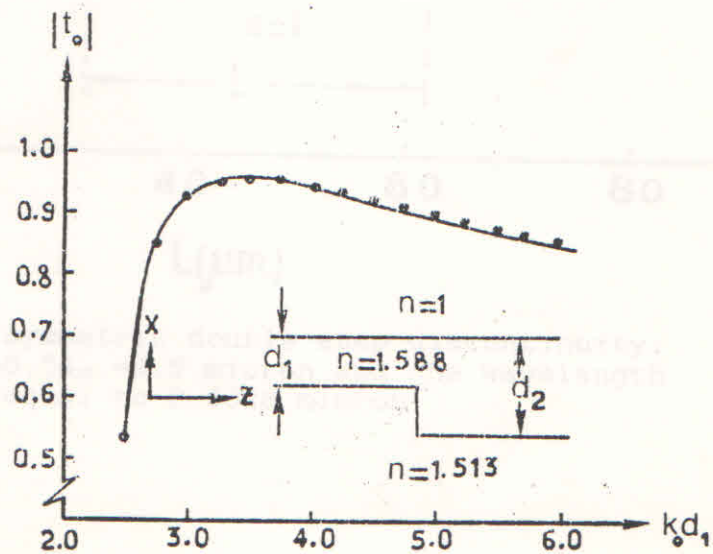——— Approximate mode matching technique
••••• BPM

Figure 2 – An asymmetric step discontinuity:
—————— Rigorous mode matching technique
••••• BPM

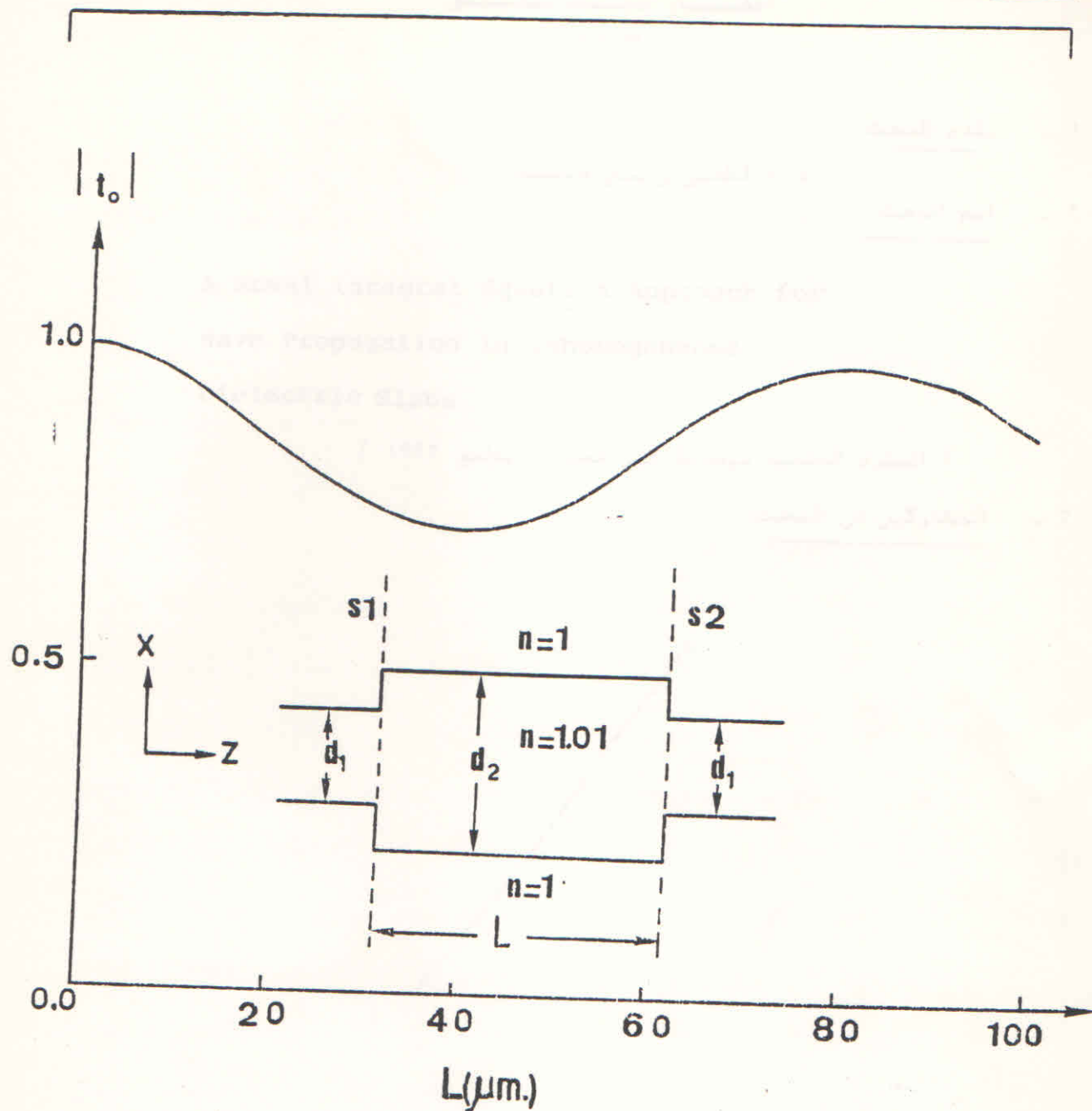Figure 3 — A symmetric double step discontinuity:
$d_1 = 0.5 d_2 = 3.5$ micron and the wavelength
is equal to 0.6328 micron.